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- OH: 1-2 pm TODAY + 1-3 pm Thursday.

TODAY:

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- Related Rates Again

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- Related Rates Again & Linear Approximation

- Drawing Graphs of Functions (including tomorrow's lecture)

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- Mean Value Theorem

Related Rates Again.

Example 1: Two people start from the same point. One walks east at 3 mi/h and the other walks northeast at 2 mi/h. How fast is the distance between the people changing after 15 minutes?

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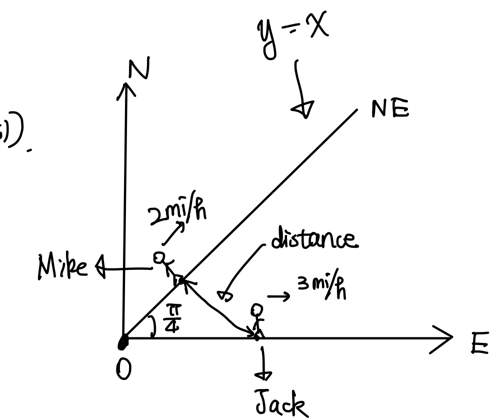
Example 1: Two people start from the same point. One walks east at 3 mi/h and the other walks northeast at 2 mi/h. How fast is the distance between the people changing after 15 minutes?



Solution. unknown: distance $d(t) = ?$

known: angle: $\frac{\pi}{4}$, position Jack: $(x_1(t), y_1(t))$, $y_1(t) = 0$

Mike: $(x_2(t), y_2(t))$, $y_2(t) = x_2$

change of position? $\Updownarrow$

$p_1(t) = \sqrt{x_1^2 + y_1^2} = \sqrt{x_1(t)^2 + 0^2} = x_1(t)$, $p_2(t) = \sqrt{x_2^2 + y_2^2} = \sqrt{x_2(t)^2 + x_2(t)^2} = \sqrt{2x_2(t)^2} = \sqrt{2}\, x_2(t)$.

$p_1'(t) = 3$ mi/h $= x_1'(t)$, $p_2'(t) = 2$ mi/h $= \sqrt{2}\, x_2'(t)$

What's $d(t)$?

Related Rates Again.

CONCLUSION: $d'(t)=\sqrt{13-6\sqrt{2}}$.

$d'(\frac{1}{4})=\sqrt{13-6\sqrt{2}}$.

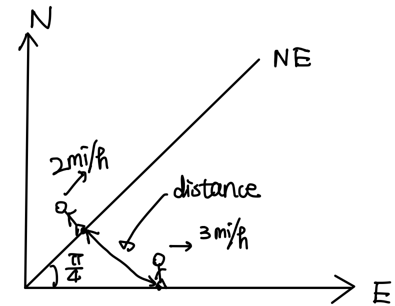
Example 1: Two people start from the same point. One walks east at 3 mi/h and the other walks northeast at 2 mi/h. How fast is the distance between the people changing after 15 minutes? $\to t=\frac{1}{4}$

Solution: known: angle: $\frac{\pi}{4}$. position Jack: $(x_1(t), y_1(t))$, $y_1(t)=0$; Mike: $(x_2(t), y_2(t))$, $y_2 = x_2$

change of position? $\sqrt{x_1(t)^2+0^2}=x_1(t)$.

$p_1(t)=\sqrt{x_1^2+y_1^2}$, $p_2(t)=\sqrt{x_2^2+y_2^2} = \sqrt{x_2(t)^2+x_2(t)^2}=\sqrt{2x_2(t)^2}=\sqrt{2}\,x_2(t)$.

$p_2=\sqrt{2}\,x_2$



$x_1'(t)=3$ mi/h $\leftarrow$ $p_1'(t)=3$ mi/h $=x_1'(t)$, $p_2'(t)=2$ mi/h $=\sqrt{2}\,x_2'(t)$

$x_2'(t)=\sqrt{2}$ mi/h

$p_2'=2$ $\quad (\sqrt{2}x_2)'=2$ $\quad \sqrt{2}\cdot x_2'=2$

$\downarrow$

$x_1(t)=3t$

$x_2(t)=\sqrt{2}t$

$$d(t)=\sqrt{(x_1(t)-x_2(t))^2+(y_1(t)-y_2(t))^2}=\sqrt{(x_1(t)-x_2(t))^2+(-x_2(t))^2}=\sqrt{x_1(t)^2-2x_1(t)x_2(t)+x_2(t)^2+x_2(t)^2}$$

$$=\sqrt{x_1(t)^2-2x_1(t)x_2(t)+2x_2(t)^2}=\sqrt{(3t)^2-2\cdot 3t\cdot\sqrt{2}t+2(\sqrt{2}t)^2}=\sqrt{9t^2-6\sqrt{2}t^2+2\cdot 2t^2}$$

$$=\sqrt{9t^2+4t^2-6\sqrt{2}t^2}=\sqrt{13t^2-6\sqrt{2}t^2}$$

$$=\sqrt{13-6\sqrt{2}}\;t.$$

Linear Approximation.

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- $f(x) = f'(a)(x-a) + o(x-a)$.

Linear Approximation.

- $f(x) = f'(a)(x-a) + \underbrace{o(x-a)}_{h(x)}$.

$$\lim_{x \to a} \frac{h(x)}{x-a} = 0.$$

- Notation o

$g(x) \overset{\text{is}}{\checkmark} o(x^n)$. $\quad \lim_{x \to 0} \frac{g(x)}{x^n} = 0$.

$g(x) = 2000x^{2n} + 1999x^{n+1}$.

Drawing Graphs

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In chapter 1 :

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In chapter 1: Start with standard models + elementary transformation

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Now: Use derivative to draw the graph.

Drawing Graphs

In chapter 1: Start with standard models + elementary transformation

Now: Use derivative to draw the graph. $y=f(x)$

(Graphing Area).

(Graphing Area)

Example 2 Draw the graph of the function $y = \dfrac{x}{x^2+1}$

Mean Value Theorem.

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- Find (abstract) extreme points

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- Estimation/Computation

(For Theorem Statement)

Rolle's. $f(a)=f(b)$, f diff in (a,b), then $\exists\, a<c<b$. $f'(c)=0$.

$-(b-a) \le \cos b - \cos a \le b-a$

Mean Value Theorem. in (a,b), f differentiable, then $\exists\, a<c<b$.

s.t.

$$f'(c) = \underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{bound for this.}}$$

$\frac{\cos b - \cos a}{b-a} = -\sin c$.

$-1 \le$

$\cos b - \cos a = (b-a)\sin c \le 1$

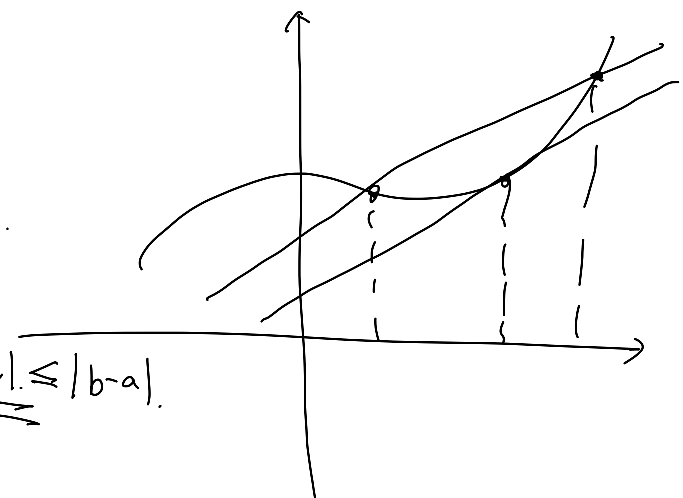
2

0.00001 $\ge$

$f(x) = \cos x$

$f'(x) = -\sin x$.

$$\left|\cos b - \cos a\right| = |b-a|\,|\sin c| \le |b-a|.$$



Example 3 If $f(1)=8$ and $f'(x)\geq 1$ for $1\leq x\leq 4$, how small can $f(4)$ possibly be?

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\;\; \uparrow \quad \uparrow$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\;\; a \quad b$

Solution.

$$\frac{f(4)-f(1)}{4-1} = f'(c), \quad 1<c<4.$$

$$\frac{f(4)-f(1)}{3} = f'(c) \geq 1$$

$$\frac{f(4)-f(1)}{3} \geq 1$$

$$\frac{f(4)-f(1)}{3} = \frac{f(4)-8}{3}$$

$$\rightsquigarrow f(4)-8 \geq 3 \rightsquigarrow f(4) \geq 3+8 = 11$$

Example 4. Show that the equation $x^4 + 4x^3 + c = 0$ has at most 2 real roots.