Oct. 19th, 2021.

Oct. 19th, 2021.

Announcement:

Oct. 19th, 2021.

## Announcement:

- midterm grading:

Oct. 19th, 2021.

## Announcement:

- midterm grading: still in progress.

Oct. 19th, 2021.

Announcement:

- midterm grading: still in progress.

- goal:

Oct. 19th, 2021.

## Announcement:

- midterm grading: still in progress.
- goal: △ write down appropriate answers;

Oct. 19th, 2021.

## Announcement:

- midterm grading: still in progress.

- goal: △ write down appropriate answers;

  △ speed up

Oct. 19th, 2021.

## Announcement:

- midterm grading: still in progress.
- goal: △ write down appropriate answers;
  △ speed up (final exam: 21 problems for 2 hours)

Oct. 19th, 2021.

## Announcement:

- midterm grading: still in progress.
- goal: △ write down appropriate answers;
  △ speed up (final exam: 21 problems for 2 hours)
- quiz:

Oct. $19^{th}$, 2021.

## Announcement:

- midterm grading: still in progress.
- goal: △ write down appropriate answers;
  △ speed up (final exam: 21 problems for 2 hours)
- quiz: • 3 problems, one from chpt 1-2.

Oct. 19th, 2021.

## Announcement:

- midterm grading: still in progress.
- goal: △ write down appropriate answers;
  △ speed up (final exam: 21 problems for 2 hours)
- quiz: • 3 problems, one from chpt 1-2. (next: related rates)

Oct. 19th, 2021.

## Announcement:

- midterm grading: still in progress.
- goal: △ write down appropriate answers;
  △ speed up (final exam: 21 problems for 2 hours)
- quiz:
  - 3 problems, one from chpt 1-2. (next: related rates)
  - 24 + 6 points

Oct. $19^{th}$, 2021.

## Announcement:

- midterm grading: still in progress.
- goal: △ write down appropriate answers;
  △ speed up (final exam: 21 problems for 2 hours)
- quiz:
  - 3 problems, one from chpt 1-2. (next: related rates)
  - 24 + 6 points (8 pts each prb, 6 bonus)

Oct. 19th, 2021.

## Announcement:

- midterm grading: still in progress.
- goal: △ write down appropriate answers;
  △ speed up (final exam: 21 problems for 2 hours)
- quiz:
  - 3 problems, one from chpt 1-2. (next: related rates)
  - 24 + 6 points (8 pts each prb, 6 bonus)
  - 24 mins.

Oct. 19th, 2021.

## Announcement:

- midterm grading: still in progress.
- goal: △ write down appropriate answers;
  △ speed up (final exam: 21 problems for 2 hours)
- quiz:
  - 3 problems, one from chpt 1-2. (next: related rates)
  - 24 + 6 points (8 pts each prb, 6 bonus)
  - 24 mins.
- OH: 1-2 pm TODAY + 1-3 pm Thursday.

TODAY:

# TODAY:

- Related Rates Again

# TODAY:

- Related Rates Again & Linear Approximation

- Drawing Graphs of Functions (including tomorrow's lecture)

# TODAY:

- Related Rates Again & Linear Approximation

- Drawing Graphs of Functions (including tomorrow's lecture)

- Mean Value Theorem

# Related Rates Again.

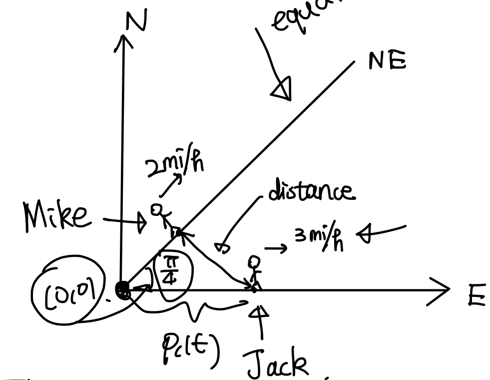
**Example 1:** Two people start from the same point. One walks east at 3 mi/h and the other walks northeast at 2 mi/h. How fast is the distance between the people changing after 15 minutes?

# Related Rates Again.

$y = y - 0 = \left(\tan\frac{\pi}{4}\right)(x-0) = 1(x-0) = x$

**Example 1:** Two people start from the same point. One walks east at 3 mi/h and the other walks northeast at 2 mi/h. How fast is the distance between the people changing after 15 minutes?

equation: $y = x$




**Solution:** Jack: $(x_1(t), y_1(t))$ $y_1 = 0$. $(x_1(t), 0)$

$p_1(t)$ = position of Jack $= \sqrt{(x_1(t)-0)^2 + (0-0)^2}$

$= \sqrt{(x_1(t))^2 + 0^2} = x_1(t)$.

$p_1'(t) = x_1'(t) = 3$ mi/h.

Mike: $(x_2(t), y_2(t))$, $p_2(t)$ = position of Mike $= \sqrt{(x_2(t))^2 + (y_2(t))^2} = \sqrt{(x_2(t))^2 + (x_2(t))^2}$

$(x_2, y_2)$ lies on $y = x$ $\rightsquigarrow$ $y_2 = x_2$.

$= \sqrt{2(x_2(t))^2} = \sqrt{2}\, x_2(t)$

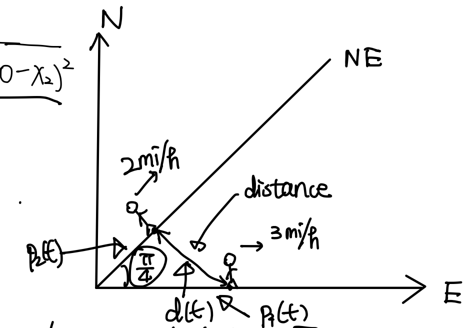
$p_2'(t) = \sqrt{2}\, x_2'(t) = 2$ mi/h.

WANT: distance between Mike & Jack $d(t) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(x_1 - x_2)^2 + (0 - x_2)^2}$

# Related Rates Again.

$$(d(t))^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2.$$

**Example 1:** Two people start from the same point. One walks east at 3 mi/h and the other walks northeast at 2 mi/h. How fast is the distance between the people changing after 15 minutes? → $\frac{1}{4}$ hour




**Solution:** WANT: distance between Mike & Jack $d(t) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(x_1 - x_2)^2 + (0 - x_2)^2}$

$$= \sqrt{(x_1(t))^2 - 2 \cdot x_1(t) \cdot x_2(t) + (x_2(t))^2 + (x_2(t))^2} = \sqrt{(x_1(t))^2 - 2x_1(t)x_2(t) + 2(x_2(t))^2}.$$

$$p_1'(t) = x_1'(t) = 3 \text{ mi/h}. \rightsquigarrow x_1(t) = 3t$$

$$p_2'(t) = \sqrt{2}\, x_2'(t) = 2 \text{ mi/h}. \rightsquigarrow x_2'(t) = \sqrt{2} \text{ mi/h} \rightsquigarrow x_2(t) = \sqrt{2}\, t.$$

$$d(t) = \sqrt{(3t)^2 - 2 \cdot 3t \cdot \sqrt{2}t + 2(\sqrt{2}t)^2} = \sqrt{9t^2 - 6\sqrt{2}t^2 + 2 \cdot 2t^2}$$

$$= \sqrt{9t^2 - 6\sqrt{2}t^2 + 4t^2} = \sqrt{(13 - 6\sqrt{2})t^2} = \sqrt{13 - 6\sqrt{2}}\, t.$$

$$d'(t) = \sqrt{13 - 6\sqrt{2}}.$$

$$d'\left(\tfrac{1}{4}\right) = \sqrt{13 - 6\sqrt{2}} \text{ mi/h}.$$

# Linear Approximation.

# Linear Approximation.

- $f(x) = f'(a)(x-a) + o(x-a)$.

# Linear Approximation.

- $f(x) = f'(a)(x-a) + o(x-a).$

- Notation $o$

# Drawing Graphs

# Drawing Graphs

In chapter 1 :

# Drawing Graphs

**In chapter 1:** Start with standard models + elementary transformation

# Drawing Graphs

**In chapter 1:** Start with standard models + elementary transformation

**Now:** Use derivative to draw the graph.

# Drawing Graphs

**In chapter 1:** Start with standard models + elementary transformation

**Now:** Use derivative to draw the graph. $y = f(x)$

(Graphing Area).

(Graphing Area)

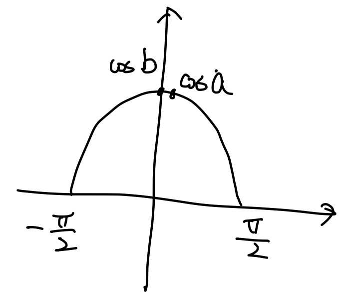
**Example 2** Draw the graph of the function $y = \frac{x}{x^2+1}$

# Mean Value Theorem.

# Mean Value Theorem.

- Find (abstract) extreme points

# Mean Value Theorem.




- Find (abstract) extreme points

- Estimation/Computation

↑ control the value of $f(x)$, $f'(x)$ (← easier to control,) use bound of $f'(x)$ to control $f(x)$.

$f(x) = \cos x$, $f'(x) = -\sin x$, $a < b \overset{\text{mean value thm.}}{\rightsquigarrow} \exists a < c < b$, $\frac{\cos b - \cos a}{b-a} = -\sin c$.

($|\cos b - \cos a| \le 2$ by the bound of $\cos x$)

$|\cos b - \cos a| = |\sin c|\,|b-a| \le |b-a|$.

$$\begin{cases} a = 10^{-40}, \; b = 0 \\ |\cos b - \cos a| \le 10^{-40} \ll 2. \end{cases}$$

(For Theorem Statement)

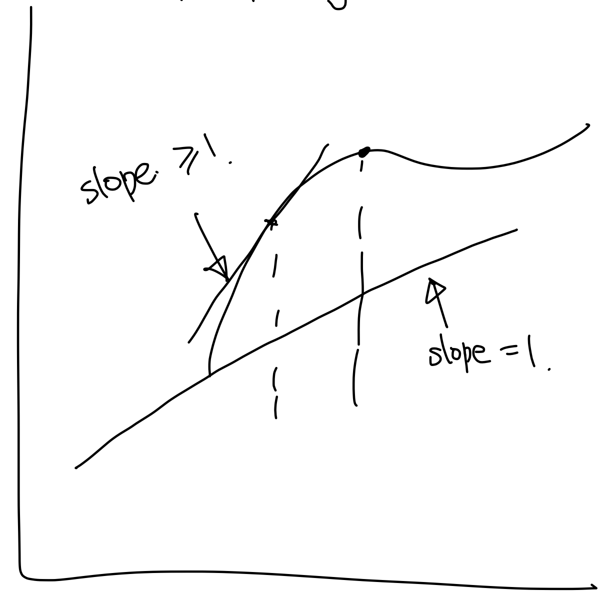
**Example 3** If $f(1)=8$ and $f'(x)\geq 1$ for $1\leq x\leq 4$, how small can $f(4)$ possibly be?

**Solution.** $\dfrac{f(4)-f(1)}{4-1} = f'(c)$, for some $1<c<4$

$\geq 1$

$\rightsquigarrow \dfrac{f(4)-8}{3} \geq 1 \rightsquigarrow f(4)-8 \geq 3$

$\rightsquigarrow f(4) \geq 3+8 = 11$ ← smallest possible value for $f(4)$.

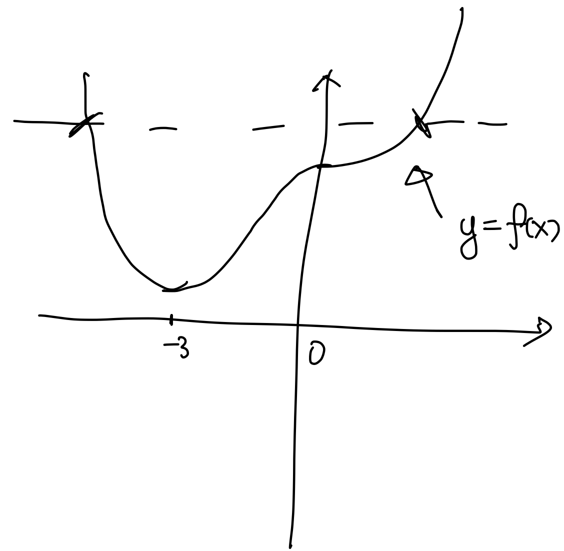


**Example 4.** Show that the equation $x^4+4x^3+c=0$ has at most 2 real roots.

**Solution:** $f(x)=x^4+4x^3+c$, $f'(x)=4x^3+12x^2=4x^2(x+3)$

$f'(x)=0 \rightsquigarrow 4x^2(x+3)=0 \rightsquigarrow \begin{cases} x_1=0 \\ x_2=-3 \end{cases}$

one local minimum. b/c we can exclude 0?




| $x<-3$, | $-3<x<0$, | $x>0$. |
|---|---|---|
| $\Uparrow$ | $\Uparrow$ | $\Uparrow$ |
| $f'(x)<0$ | $f'(x)>0$ | $f'(x)>0$. |
| | $\Downarrow$ | $\Downarrow$ |
| $\rightsquigarrow f(x)$ decreasing. | increasing | increasing. |

$\rightsquigarrow$ at most 2 roots.