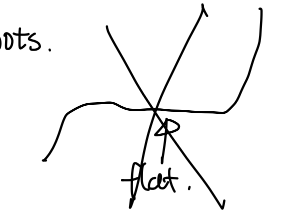
Oct. 26th, 2021

Curve Sketching
&
Optimization Problem

Mean Value Theorem (10 am)

Problem. Show that equation $\underset{f(x)}{\underbrace{x^4+4x^3+c}}=0$ has at most 2 real roots.



which MVT we nee to use? Rolle's theorem,

$f'(x) = 4x^3+12x^2$

$= 4x^2(x+3)$.

$f'(x)=0 \Longleftrightarrow x_1=0,\ x_2=-3$

$\begin{cases} \underline{x<-3},\ f'(x)<0. \\ x>-3,\ f'(x)\geq 0 \end{cases}$

$\Rightarrow f$ decreasing in $(-\infty,-3)$.

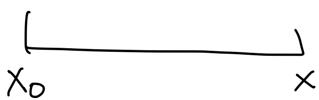
increasing in $[-3,+\infty)$.

$\rightsquigarrow f$ has at most 1 root in $(-\infty,-3)$

& at most 1 root in $(-3,+\infty)$.

$\overset{0}{\underset{\|}{}}$

$\underline{x_0<}c<x_1$ impossible!



$x_0 = 0$.

pick $x_1=0$.

if $x<-3$, $\underline{x+3<0}$, $x^2\geq 0$. $\overset{f'(x).}{4x^2(x+3)}<0$.

$x>-3$, $x+3>0$. $x^2\geq 0$ $4x^2(x+3)>0$.

pick $x_1>x_0$. Apply MVT.

$$\frac{f(x_1)-f(x_0)}{x_1-x_0} = \underline{f'(c)}\geq 0.$$

$f(x_1)\geq f(x_0)$.

$x_0<0$

then

$f(x_0)<f(x_1)=f(0)$

pick $x_0=0$,

$f(0)<f(x_1)$

$\|$

$f(x_0)$.

Mean Value Theorem (10 am)

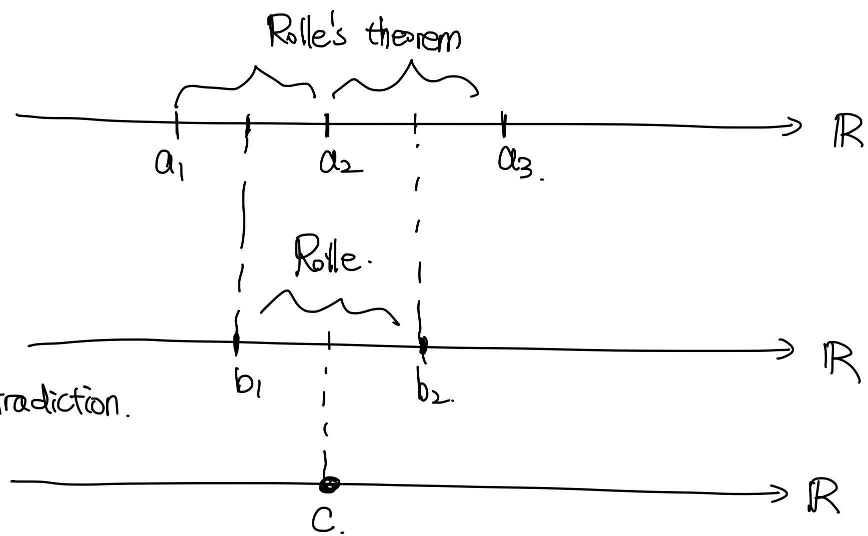
Problem. Show that equation $x^4+4x^3+c=0$ has at most 2 real roots.

$\rightsquigarrow$ f has at most 2 roots

$$x^4+4x+c=0.$$

Assume this equation has at least 3 distinct roots. $a_1<a_2<a_3$,

$x^4+4x+c = f(x)$



$x^3+1=0.$

$4(x^3+1) = 4x^3+4 = f'(x)$

has exactly 1 root -1. $\rightsquigarrow$ contradiction.

$f''(x).$

Review:

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Question: What things we should determine when we draw a graph?

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domain & range.

asymptotics : horizontal & vertical. $\lim\limits_{\substack{x\to+\infty \\ -\infty}} f(x)$, & where $\lim\limits_{x\to a} f(x)=\pm\infty$

monotonicity : $f'(x)>0$

$f'(x)<0$.

convexity : $f''(x)>0$

$f''(x)<0$.

special points: critical pts. $\rightsquigarrow$ local maximum & local minimum.

inflection pts.

x- and y-intersects.

Curve Sketch

Problem 2 Sketch the graph of the function $y = x + \frac{1}{x} + 1. = f(x)$

domain: $(-\infty, 0) \cup (0, +\infty)$

asymptotes: vertical asymptote @ $x=0$. $\lim_{x \to 0^-} \left(x + \frac{1}{x} + 1\right) = -\infty$, and $\lim_{x \to 0^+} \left(x + \frac{1}{x} + 1\right) = +\infty$.

no horizontal asymptote.

monotonicity: decreasing in $(0,1)$ and increasing in $(1, +\infty)$.

increasing in $(-\infty, -1)$ and decreasing in $(-1, 0)$.

$y' = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$

$y' > 0$ or $y' < 0$.

$\Downarrow$ $\Downarrow$

$x^2 - 1 > 0$, $x^2 - 1 < 0$.

$\Downarrow$ $\Downarrow$

$x > 1$ or $x < -1$ $-1 < x < 1$

$(-\infty, -1) \cup (1, +\infty)$. $x \neq 0$.

$(-1, 0) \cup (0, 1)$

convexity: convex $(f''(x) > 0)$. in $(0, +\infty)$ concave $(f''(x) < 0)$. in $(-\infty, 0)$.

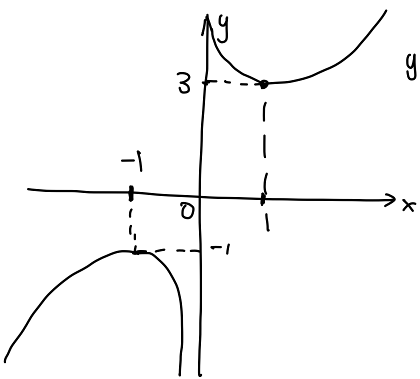
critical pts: $x_1 = -1$, $x_2 = 1$. $\longrightarrow$ local minimum. $f(1) = 3$.

$x_1 = -1 \longrightarrow$ local maximum $f(-1) = -1$.

inflection pts: NO

x- and y-intersects: NO.

graph:



$y = x + \frac{1}{x} + 1$.

$y'' = \frac{2}{x^3}$,

Curve Sketch

Problem 2 Sketch the graph of the function $y = x + \frac{1}{x} + 1$.

$a > 0$
$b > 0$.

$\sqrt{a \cdot b} \le \frac{a+b}{2}$. put $a = x$ and $b = \frac{1}{x}$.

$\rightsquigarrow \sqrt{x \cdot \frac{1}{x}} \le \frac{1}{2}\left(x + \frac{1}{x}\right)$. $\rightsquigarrow$ $x + \frac{1}{x} \geqslant 2\sqrt{1} = 2$.

$x + \frac{1}{x} + 1 \geqslant 3$ if $x > 0$.

$x < 0$: $a = -x$ and $b = \frac{-1}{x}$

$\sqrt{(-x) \cdot \left(\frac{-1}{x}\right)} \le \frac{1}{2}\left(-x - \frac{1}{x}\right)$. $\rightsquigarrow$ $-\left(x + \frac{1}{x}\right) \geqslant 2$.

$1 + x + \frac{1}{x} \le -2 + 1 = -1$.

when $x < -1$.

Optimization Problem

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Applications of things you learn in chapter 3.

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Problem 3. A farmer has 1600 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

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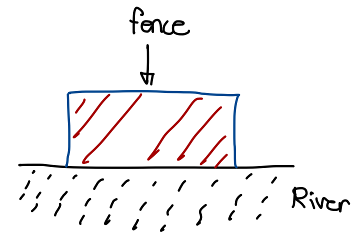
Applications of things you learn in chapter 3.

Problem 3. A farmer has 1600 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?



Reminder:

- Quiz on Thursday. (Practice Problem: 27 of Section 3.2, 17-18 of Section 3.2, and 4 of Section 2.7)
- Office hours: 1-2 pm TODAY (Other times by appointment)
- See you on Thursday!