Oct. 26th, 2021

Curve Sketching

&

Optimization Problem

Mean Value Theorem (10 am)

Problem. Show that equation $x^4+4x^3+c=0$ has at most 2 real roots.

Review:

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Question: What things we should determine when we draw a graph?

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domain: $\frac{x-1}{x^2-1}$. $(-\infty,-1)\cup(-1,1)\cup(1,+\infty)$.

asymptotics: $\lim_{x\to\pm\infty} f(x)$, $\lim_{x\to a^{-+}} f(x)=\pm\infty$.

monotonicity: $f(x)$ increasing or decreasing.

$f'(x)>0$. $f'(x)<0$.

convexity: $f(x)$ convex or concave.

$f''(x)>0$. $f''(x)<0$.

special points: x-intersects & y-intersects.

critical points.

inflection points.

Curve Sketch

Problem 2 Sketch the graph of the function $y = x + \frac{1}{x} + 1. = f(x)$

domain: $(-\infty, 0) \cup (0, +\infty)$.

asymptotes: $x = 0$ vertical asymptote.

NO horizontal asymptotes.

$$f'(x) = -\frac{1}{x^2} + 1,$$
$$= \frac{x^2 - 1}{x^2}$$

$f'(x) > 0$ $(1, +\infty) \cup (-\infty, -1)$.

$f'(x) < 0$. $(-1, 0) \cup (0, 1)$.

$f'(x) = 0$: $+1, -1$

$f(1) = 3$

$f(-1) = -1$

$\frac{x^2-1}{x^2} > 0 \Rightarrow x^2 - 1 > 0$.

$x^2 > 1$

$\Rightarrow x < -1$ or $x > 1$.

$\frac{x^2-1}{x^2} < 0 \Rightarrow x^2 < 1$

$\Rightarrow -1 < x < 1$ $x \neq 0$.

$$f''(x) = \frac{2}{x^3},$$

$f''(x) = 0$: NO.

$f''(x) > 0$: $(0, +\infty)$

$f''(x) < 0$: $(-\infty, 0)$

x-intersects: NO?

y-intersects: NO.

$x = -1$, $f''(x) < 0 \rightsquigarrow f(-1)$ is a local maximum; $x = +1$, $f''(x) > 0$, $f(1)$ is a local minimum.

Curve Sketch

Problem 2 Sketch the graph of the function $y = x + \frac{1}{x} + 1$.

$f'(x) = -\frac{1}{x^2} + 1,$

$= \frac{x^2-1}{x^2}$

$f'(x) > 0 \quad (1, +\infty) \cup (-\infty, -1).$

$f'(x) < 0. \quad (-1, 0) \cup (0, 1).$

$f'(x) = 0 : \quad +1, -1$

$f(1) = 3$

$f(-1) = -1$

$\frac{x^2-1}{x^2} > 0 \Rightarrow x^2 - 1 > 0.$

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$f''(x) = 0 :$ NO.

$f''(x) > 0 : (0, +\infty)$

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x-intersects : NO?

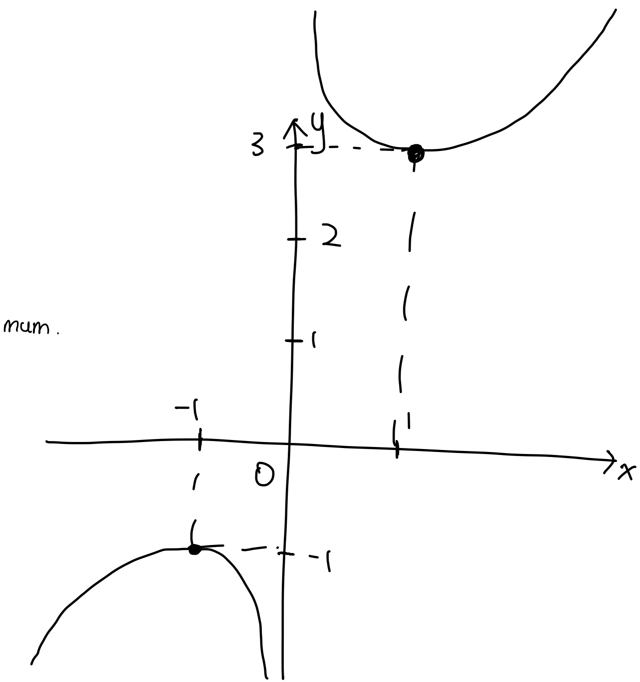
y-intersects : NO.

$x = -1$, $f''(x) < 0 \rightsquigarrow f(-1)$ is a local maximum; $x = +1$, $f''(x) > 0$, $f(1)$ is a local minimum.

for any $x < 0$, $f(x) \leq f(-1) = -1 < 0.$

for any $x > 0$, $f(x) \geq f(1) = 3 > 0.$

$\rightsquigarrow f$ has no roots $\rightsquigarrow f$ has no x-intersects.



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Applications of things you learn in chapter 3.

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Problem 3. A farmer has 1600 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

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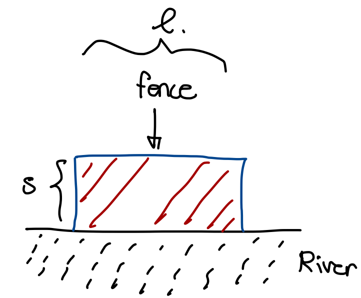

s & ℓ. $A = s\ell$

$\ell + 2s = 1600$ ft

$2s \le 1600$.

$s \le 800$

find maximum of A as s & ℓ varies.

$\ell = 1600 - 2s$

$A = s\ell = (1600-2s)s = -2s^2 + 1600s = A(s)$ what's domain of A? $[0, 800]$.

$A'(s) = -4s + 1600$ $A'(s) = 0$. when $s = 400$ ft. $s=400$ is a local maximum.

$A''(s) = -4 < 0$

$\ell = 1600 - 2s$
$= 800$ ft.

$A(400) = -2 \times 400^2 + 1600 \times 400$
$= 640000 - 2 \times 160000$
$= 320000 \text{ ft}^2$.

Is $s=400$ an absolute maximum? ✓

Reminder:

- Quiz on Thursday. (Practice Problem: 27 of Section 3.2, 17-18 of Section 3.2, and 4 of Section 2.7)

- Office hours: 1-2 pm TODAY (Other times by appointment)

- See you on Thursday!