Nov. 11^{th} 2021

Review Before Midterm

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Today:

- Curve Sketching Problem Again
- Mean Value Theorem Again
- Riemann Sum Again.

Curve Sketching Again

given a function $y=f(x)$, sketching the graph of f.

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Things to determine:

- domain.
- critical pts (maximum, minimum), first derivative $f'(x)$.
- second derivative & inflection pts.
- convexity.
- asymptotes. (horizontal & vertical).
- increasing & decreasing.
- roots ($f(x)=0$). $\rightarrow$ x-intersect & y-intersect ($f(0)$).

Curve Sketching Again

given a function $y=f(x)$, sketching the graph of f.

Things to determine:

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-
-

Actually you don't need to remember all of them.

Example.

Problem 5. Consider the function $f(x) = \dfrac{\sqrt{4-x^2}}{x+1}$ on the domain $[-2,-1) \cup (-1,2]$. You may freely use any of the following facts.

$f(x) = \dfrac{\sqrt{4-x^2}}{x+1}$.

$[-2,-1) \cup (-1,2]$

i) $f'(x) = \dfrac{-x-4}{(x+1)^2\sqrt{4-x^2}}$

ii) $f''(x) = \dfrac{(x+1.84)(1.38-x)K(x)}{x+1}$, where $K(x) > 0$

iii) $f(1.38) = 0.6$

iv) $f(-1.84) = -0.93$

a) Study the sign of $f'(x)$. Determine the intervals where f is increasing, and the intervals where it is decreasing. Indicate the values of the local extrema, if any. You must justify your findings.

$f'(x) = \dfrac{-x-4}{(x+1)^2\sqrt{4-x^2}}$ $\quad f'(x) = 0 \quad \dfrac{-x-4}{\underbrace{(x+1)^2\sqrt{4-x^2}}_{\neq 0}} = 0$ needs $-x-4=0$, $x=-4$. is NOT in the domain of f.

f has NO critical pts in $(-2,-1) \cup (-1,2)$, but $f'(-2)$ and $f'(2)$ DNE. (2 & -2 are critical pts of f.)

$x > -4$, then $-x < 4$, so $-x-4 < 0$. $\sqrt{4-x^2} \geq 0$, $(x+1)^2 \geq 0$, so

$$\frac{-x-4}{(x+1)^2\sqrt{4-x^2}} < 0$$, that is, f is decreasing in $[-2,-1) \cup (-1,2]$.

$f(-2) = f(2) = 0$

b) Study the sign of $f''(x)$. Determine the intervals where f is concave up, and the intervals where it is concave down. List the inflection points, if any. You must justify your findings.

$f''(x) = \frac{(x+1.84)(1.38-x)K(x)}{x+1}$ where $K(x) > 0$.

inflection pt: $f''(c)=0$ or $f''(c)$ DNE.

$f''(x)=0 \Leftrightarrow \frac{(x+1.84)(1.38-x)K(x)}{x+1} = 0$, but $K(x) \neq 0$ & $x+1 \neq 0$ if f'' is well-defined, so.

$(x+1.84)(1.38-x)=0$ has two roots: -1.84, 1.38.

$f''(x)$ DNE: $x+1=0$, $x=-1$, but -1 does not lie in the domain of f. So -1 is NOT the inflection pt of f.

Two inflection pts: -1.84, 1.38, $f(1.38) \approx 6.6$, $f(-1.84) = -0.93$.

$[-2,-1.84) \cup (-1.84,-1) \cup (-1,1.38) \cup (1.38,2]$.

① when $-2 \le x < -1.84$, then $x+1.84<0$, $1.38-x>0$, $x+1<0$ because $x<-1.84<-1$. $\rightsquigarrow f''(x)>0$.

② when $-1.84<x<-1$, then $x+1.84>0$, $1.38-x>0$, $x+1<0$ $\rightsquigarrow f''(x)<0$.

③ when $-1<x<1.38$, then $x+1.84>0$, $1.38-x>0$, $x+1>0$. $\rightsquigarrow f''(x)>0$.

④ $1.38<x\le 2$, then $x+1.84>0$. $+$
$1.38-x<0$. $-$
$x+1>0$. $+$
$\rightsquigarrow f''(x)<0$.

c) Investigate for the existence of vertical/horizontal asymptotes. Your findings must be supported by the careful calculation of relevant limits. (Each vertical asymptote must be supported by two limits.)

$f(x)=\frac{\sqrt{4-x^2}}{x+1}$.

$[-2,-1)\cup(-1,2]$

i) $f'(x) = \dfrac{-x-4}{(x+1)^2\sqrt{4-x^2}}$

ii) $f''(x) = \dfrac{(x+1.84)(1.38-x)K(x)}{x+1}$, where $K(x) > 0$

iii) $f(1.38) = 0.6$

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NO horizontal asymptotes.

vertical asymptotes: $x+1=0$, $x=-1$.

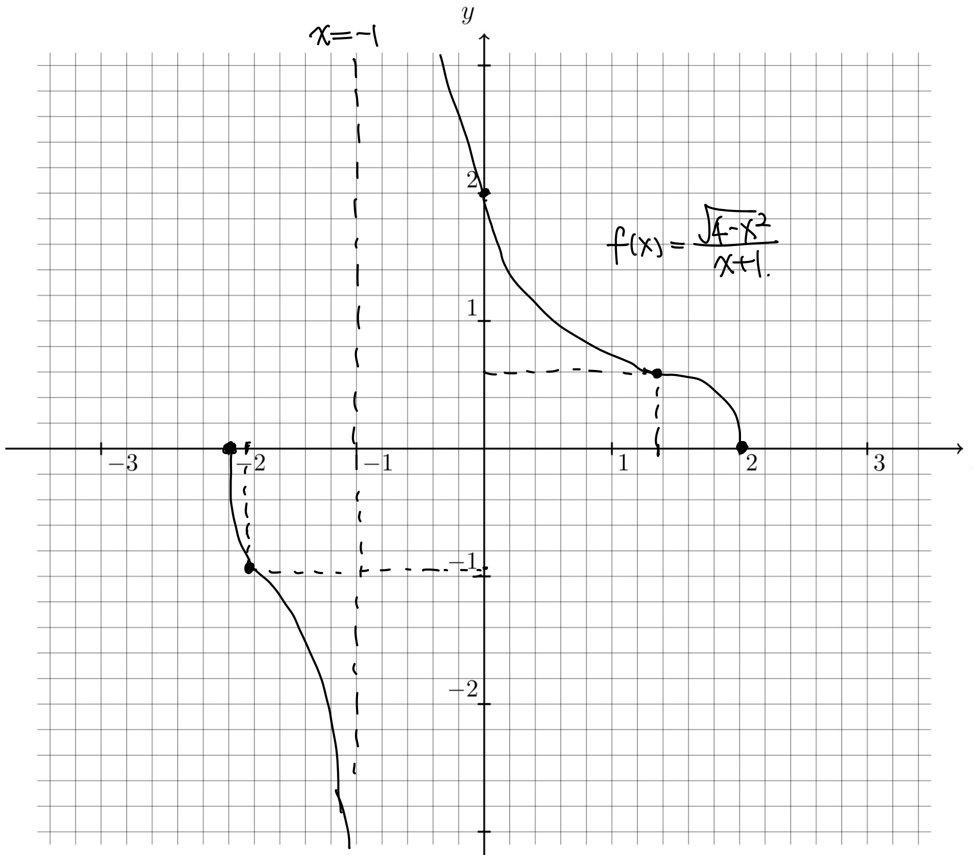
$$\boxed{\lim_{x\to -1^+} \frac{\sqrt{4-x^2}}{x+1} = +\infty \qquad \lim_{x\to -1^-} \frac{\sqrt{4-x^2}}{x+1} = -\infty.}$$

$x=-1$ is a vertical asymptote.

$$f(0) = \frac{\sqrt{4-0^2}}{0+1} = \frac{\sqrt{4}}{1} = 2$$

d) Based on all the information gathered in the previous questions, sketch the graph of f as accurately as possible. Include and clearly label all relevant points and asymptotes.

$-1.84 > -2$



Mean Value Theorem Again.

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Apply mean value theorem to prove inequalities.

Mean Value Theorem Again.

Apply mean value theorem to prove inequalities.

Example. **Problem 7.** One wants to calculate the value of the limit

$$\ell = \lim_{x\to 0^+} \frac{\sqrt[3]{5x+64}-4}{\sqrt[5]{x}}.$$

a) Show that $\sqrt[3]{5x+64} < 4 + \frac{5}{48}x$ for all $x > 0$. You must justify your methods.

$\frac{f(x+h)-f(x)}{x+h-x} = f'(c)$ for some $x<c<x+h$.

$x+h-x = h.$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f(x+h)-f(x) = f'(c)\cdot h$.

$f(y) = \sqrt[3]{y}$. let $y=64$, &

$4 = \sqrt[3]{64}$ $\quad 4^3 = 64.$

$\frac{f(5x+64)-f(64)}{5x+64-64}$ exists $64<c<64+5x$.

$$\frac{\sqrt[3]{5x+64}-\sqrt[3]{64}}{5x+64-64} = f'(c) = \frac{1}{3\sqrt[3]{c^2}} < \frac{1}{3\cdot\sqrt[3]{64}^2} = \frac{1}{3\cdot 4^2} = \frac{1}{48}.$$

Riemann Sum Again

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Write down the definite integral a Riemann sum represent.

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Write down the definite integral a Riemann sum represent.

Example. $$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{2}{n}}+\cdots+\sqrt{1+\frac{n}{n}}\right) = \int$$

Reminder:

- 2^{nd} midterm next week: 2×50 mins
- Contents: Chapter 3 for part I (T)

  Chapter 4 for part II (Th)
- Classroom: TBD
- Index card of notes again
- OH: 3-4 today. Appointment accepted

  Next Week: Mon 5-6 pm

  Wed 3-5 pm