Nov. 11^{th} 2021

Review Before Midterm

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Today:

- Curve Sketching Problem Again
- Mean Value Theorem Again
- Riemann Sum Again.

Curve Sketching Again

given a function $y=f(x)$, sketching the graph of f.

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Things to determine:

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Curve Sketching Again

given a function $y=f(x)$, sketching the graph of f.

Things to determine:

- domain
- asymptotes.
- critical pts, first derivative, & increasing/decreasing.
- inflection pts, second derivative, & convex/concave.
- x-intersects & y intersects.
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Actually you don't need to remember all of them.

Example.

Problem 5. Consider the function $f(x) = \frac{\sqrt{4-x^2}}{x+1}$ on the domain $[-2,-1) \cup (-1,2]$. You may freely use any of the following facts.

i) $f'(x) = \dfrac{-x-4}{(x+1)^2\sqrt{4-x^2}}$

ii) $f''(x) = \dfrac{(x+1.84)(1.38-x)K(x)}{x+1}$, where $K(x) > 0$

iii) $f(1.38) = 0.6$

iv) $f(-1.84) = -0.93$

a) Study the sign of $f'(x)$. Determine the intervals where f is increasing, and the intervals where it is decreasing. Indicate the values of the local extrema, if any. You must justify your findings.

$f'(x) = \dfrac{-x-4}{(x+1)^2\sqrt{4-x^2}} = 0$, then $x=-4$ which is not in the domain of f.

$f'(x)$ DNE if $(x+1)^2\sqrt{4-x^2} = 0$. $(x+1)^2 = 0$ or $\sqrt{4-x^2} = 0 \rightsquigarrow x = \pm 2$ & they're critical pts of f.

$x=-1 \to$ NOT a critical pt

for any x inside domain of f, $x > -4$, then $-x-4 < -(-4)-4 = 4-4 = 0$

so $f'(x) < 0$ for any $-2 \leq x \leq 2$ and $x \neq -1$. So f is decreasing.

$f(-2) = f(2) = 0$.

b) Study the sign of $f''(x)$. Determine the intervals where f is concave up, and the intervals where it is concave down. List the inflection points, if any. You must justify your findings.

$$f''(x) = \frac{(x+1.84)(1.38-x)K(x)}{x+1}, \quad \text{where } K(x)>0.$$

Inflection points: $\begin{cases} f''(x)=0 \rightsquigarrow (x+1.84)(1.38-x)=0. \text{ solutions: } x_1=-1.84 \text{ and } x_2=1.38. \\ \text{or} \\ f''(x) \text{ DNE } \quad x+1=0, \ x=-1 \text{ does not lie in the domain of } f. \end{cases}$

sign of f'': $[-2, -1.84) \cup (-1.84, -1) \cup (-1, 1.38) \cup (1.38, 2]$.

1. $-2 \le x < -1.84$: $x+1.84<0$, $1.38-x>0$, $x+1<0$ $\rightsquigarrow f''(x)>0$ concave up
2. $-1.84 < x < -1$: $x+1.84>0$, $1.38-x>0$, $x+1<0$ $\rightsquigarrow f''(x)<0$ concave down. $\Rightarrow -1.84$ is an inflec pt.
3. $-1<x<1.38$ $x+1.84>0$, $1.38-x>0$, $x+1>0$ $\rightsquigarrow f''(x)>0$ concave up
4. $1.38<x\le 2$ $x+1.84>0$, $1.38-x<0$, $x+1>0$ $\rightsquigarrow f''(x)<0$ concave down. 1.38 is an inf. pt.

$f(-1.84) = -0.93.$

$f(1.38) = 0.6.$

c) Investigate for the existence of vertical/horizontal asymptotes. Your findings must be supported by the careful calculation of relevant limits. (Each vertical asymptote must be supported by two limits.)

$$f(x) = \frac{\sqrt{4-x^2}}{x+1}.$$

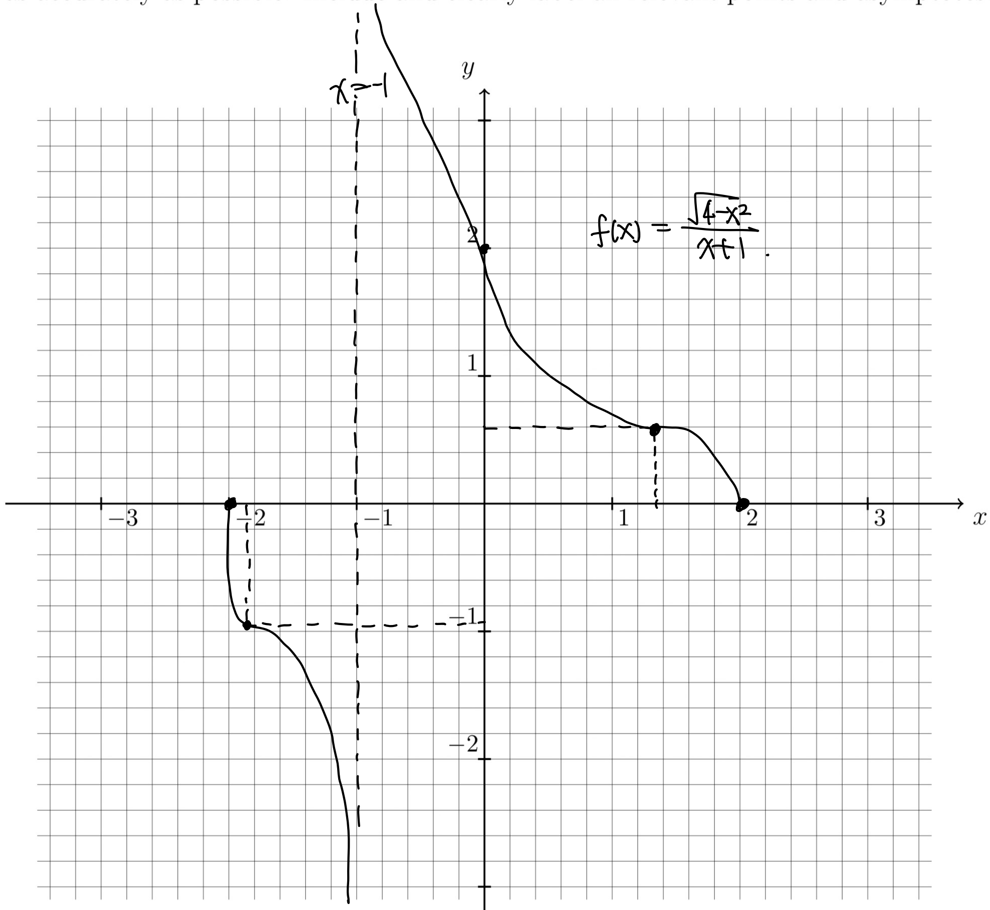
No horizontal asymptotes because $\lim_{x\to+\infty} f$ & $\lim_{x\to-\infty} f$ makes no sense (domain is $[-2,1)\cup(1,2]$).

vertical asymptotes : denominator $=0$: $x+1=0$, $x=-1$.

$$\lim_{x\to -1^-} f(x) = \lim_{x\to 1^-} \frac{\sqrt{4-x^2}}{x+1} = -\infty; \quad \lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} \frac{\sqrt{4-x^2}}{x+1} = +\infty.$$

$x=-1$ is a vertical asymptote.

d) Based on all the information gathered in the previous questions, sketch the graph of f as accurately as possible. Include and clearly label all relevant points and asymptotes.



$$f(0) = \frac{\sqrt{4-0^2}}{0+1}$$

$$= 2.$$

Mean Value Theorem Again.

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Apply mean value theorem to prove inequalities.

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Apply mean value theorem to prove inequalities.

Example. (Spring 2019) **Problem 7.** One wants to calculate the value of the limit

$$\ell = \lim_{x\to 0+} \frac{\sqrt[3]{5x+64}-4}{\sqrt[5]{x}}.$$

$\frac{1}{3}-1=-\frac{2}{3}$.

$(x^{\frac{1}{3}})' = \frac{1}{3}x^{\frac{1}{3}-1}$

a) Show that $\sqrt[3]{5x+64} < 4 + \frac{5}{48}x$ for all $x > 0$. You must justify your methods.

27. Show that $\sqrt{1+x} < 1 + \frac{1}{2}x$ if $x > 0$.

$f(x) = \sqrt{x}$

$\frac{f(1+x)-f(1)}{1+x-1} = f'(c)$ for some $1<c<1+x$.

then $\sqrt{c}>1$

$f'(x) = \frac{1}{2\sqrt{x}}$, $f'(c) = \frac{1}{2\sqrt{c}} < \frac{1}{2}$.

$\frac{1}{\sqrt{c}} < 1$

so we get $\frac{\sqrt{1+x}-\sqrt{1}}{x} < \frac{1}{2}$

$\sqrt{1+x}-1 < \frac{1}{2}x$, $\rightsquigarrow \sqrt{1+x} < 1+\frac{1}{2}x$.

$f(x) = \sqrt[3]{x}$, $f'(x) = \frac{1}{3\sqrt[3]{x^2}}$.

$\sqrt[3]{5x+64}-4 = \sqrt[3]{5x+64}-\sqrt[3]{64}$ $(4^3=64)$.

$= f(5x+64) - f(64)$.

Apply MVT: where $64<c<5x+64$.

$\frac{f(5x+64)-f(64)}{5x+64-64} = f'(c) = \frac{1}{3\sqrt[3]{c^2}}$.

$c>64$, then $\sqrt[3]{c}>4$, $\sqrt[3]{c^2} > 4^2 = 16$

$\frac{1}{3\sqrt[3]{c^2}} < \frac{1}{3\cdot 16} = \frac{1}{48}$.

Riemann Sum Again

finally, $\dfrac{\sqrt[3]{5x+64}-4}{5x} < \dfrac{1}{48}$

$$\sqrt[3]{5x+64} - 4 < \frac{5}{48}x.$$

$$\sqrt[3]{5x+64} < 4 + \frac{5x}{48}$$

Riemann Sum Again

Write down the definite integral a Riemann sum represent.

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Write down the definite integral a Riemann sum represent.

Example. $$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{2}{n}}+\cdots+\sqrt{1+\frac{n}{n}}\right)=\int$$

Reminder:

- 2^{nd} midterm next week: 2×50 mins
- Contents: Chapter 3 for part I (T)

  Chapter 4 for part II (Th)
- Classroom: TBD
- Index card of notes again
- OH: 3-4 today. Appointment accepted

  Next Week: Mon 5-6 pm

  Wed 3-5 pm