14. y' for $y=\int_{\sin x}^{\cos x}(1+v^2)^{10}\,dv$.

fundamental thm of calculus $F(t)=\int_0^x f(t)\,dt$, then $f(x)=F'(x)$.

$y=\int_0^{\cos x}(1+v^2)^{10}\,dv-\int_0^{\sin x}(1+v^2)^{10}\,dv$. write $f(v)=(1+v^2)^{10}$, then

$F(w)=\int_0^w f(v)\,dv$.

$y=F(\cos x)-F(\sin x)$. use chain rule:

$y'=F'(\cos x)\cdot(\cos x)'-F'(\sin x)\cdot(\sin x)'$.

$=f(\cos x)\cdot(-\sin x)-f(\sin x)\cdot\cos x$.

by FTC

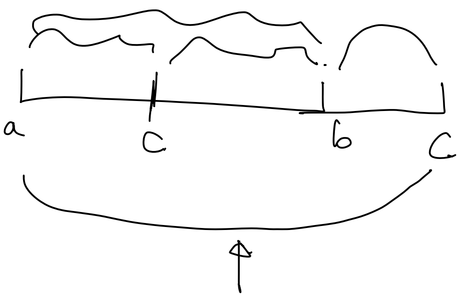
$=-(1+\cos^2 x)^{10}\sin x-(1+\sin^2 x)^{10}\cdot\cos x$.

$\int_a^b f(x)\,dx$

for any real number c,

$\int_a^b f(x)\,dx$

$=\int_c^b f(x)\,dx+\int_a^c f(x)\,dx$



30. $Si(x)=\int_0^x \frac{\sin t}{t}\,dt$. $Si(x)$ is defined in $\mathbb{R}$. $Si(0)=0$.

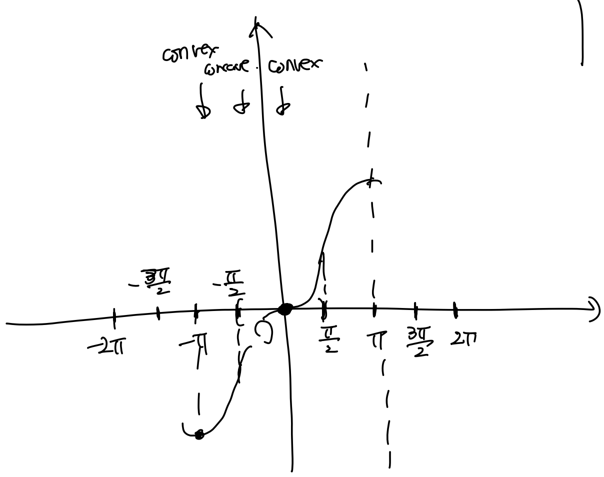
(a) Draw the graph.

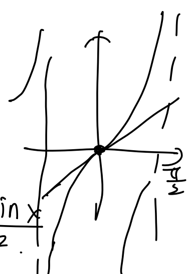
$\boxed{Si'(x)=\frac{\sin x}{x}}$ FTC

$Si'(x)>0$: $x>0$: $\sin x>0$, $[0,\pi)$, $(2\pi,3\pi)$, ...
$x<0$: $\sin x<0$, $(-\pi,0]$, $(-3\pi,-2\pi)$, ...

$Si'(x)=0$: $x=\pi,2\pi,3\pi,\cdots$
$x=-\pi,-2\pi,-3\pi,\cdots$

$Si'(x)<0$: $x>0$: $(\pi,2\pi)$, $(3\pi,4\pi)$, ...
$x<0$: $(-2\pi,-\pi)$, $(-4\pi,-3\pi)$, ...





$Si''(x)=\frac{x\cos x-\sin x}{x^2}$.

$(\tan x)'=\sec^2 x$

$\sec^2 0=1$

$x\cos x-\sin x>0$.

$x\cos x>\sin x$

if $\cos x>0$, $\boxed{x>\tan x}$

if $\cos x<0$, $x<\tan x$.

31. Find $f(x)$ and a s.t.

$6+\int_a^x \frac{f(t)}{t^2}\,dt=2\sqrt{x}$ for any x.

FTC

Take derivative: $\frac{f(x)}{x^2}=\frac{1}{\sqrt{x}}$. $\Rightarrow f(x)=x^2\cdot\frac{1}{\sqrt{x}}=x^{\frac{3}{2}}$.

$\boxed{6+\int_a^x \frac{t^{\frac{3}{2}}}{t^2}\,dt=2\sqrt{x}}$.

$2\sqrt{x}-2\sqrt{a}+6=6+\int_a^x \frac{t^{\frac{3}{2}}}{t^2}\,dt=2\sqrt{x}$

$$6+\int_a^x \frac{1}{\sqrt{t}}\,dt = 6+2\sqrt{t}\Big|_a^x = 2\sqrt{x}-2\sqrt{a}+6$$

$$2\sqrt{x}-2\sqrt{a}+6 = 2\sqrt{x}.$$

so $6-2\sqrt{a}=0$, and $\sqrt{a}=3$, $a=9$.

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$.

Sec. 4.3, 40. $\int \cos^2 x\,dx = \underbrace{\frac{1}{2}x + \frac{1}{4}\sin 2x + C}_{F(x)}$.

$$F'(x) = \frac{1}{2} + \frac{1}{2}\cdot(2\cos 2x) = \frac{1}{2} + \frac{1}{2}\cos 2x.$$

$$= \frac{1}{2} + \frac{1}{2}(2\cos^2 x - 1)$$

$$= \frac{1}{2} + \left(\frac{1}{2}\cdot 2\cos^2 x\right) - \frac{1}{2}$$

$$= \cos^2 x.$$