2. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

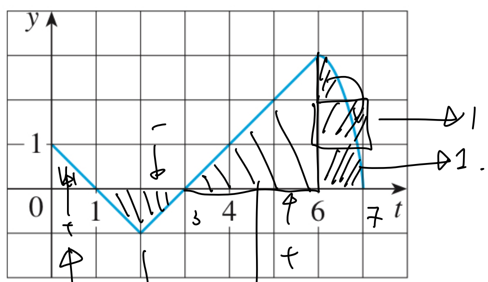
(a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5,$ and 6.

(b) Estimate $g(7)$.

(c) Where does g have a maximum value? Where does it have a minimum value?

(d) Sketch a rough graph of g.

$g(7)$



$\frac{1}{2} - 1 + \frac{9}{2} + 2 = 5+2-1 = 6.$

CAS 30. The **sine integral function**

$$\text{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt$$

is important in electrical engineering. [The integrand $f(t) = (\sin t)/t$ is not defined when $t = 0$, but we know that its limit is 1 when $t \to 0$. So we define $f(0) = 1$ and this makes f a continuous function everywhere.]

(a) Draw the graph of Si.

(b) At what values of x does this function have local maximum values?

(c) Find the coordinates of the first inflection point to the right of the origin.

(d) Does this function have horizontal asymptotes?

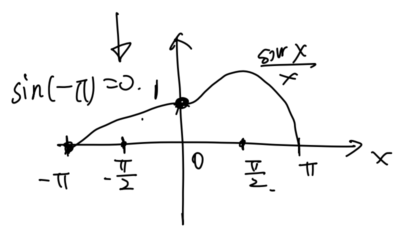
(e) Solve the following equation correct to one decimal place:

$$\int_0^x \frac{\sin t}{t}\,dt = 1$$

$\text{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt.$

$\text{Si}'(x) = \frac{\sin x}{x}.$

$\sin(-\pi) = 0.$



$\text{Si}''(x) = \frac{x\cos x - \sin x}{x^2}.$

$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1.}$

$x\cos x - \sin x = 0.$

$x_1 = 0,$

$x = \tan x.$

$x\cos x - \sin x > 0$

$x > \tan x$

as $x < +\frac{\pi}{2}$

$(\tan x)' = \sec^2 x$

$x > \tan x$ then $x\cos x -$

$\sec^2 0 = 1.$

$\sec^2 x > 1$ as $x \neq 0.$

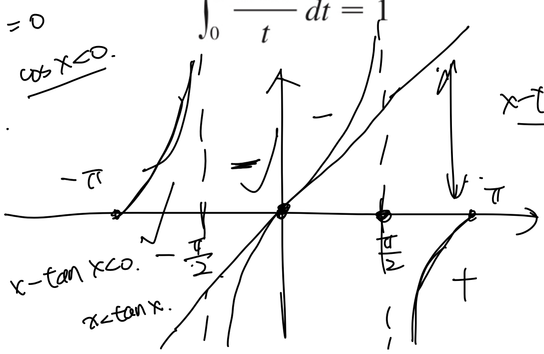
$(x - \tan x)' \geq 0$ in $(-\frac{\pi}{2}, \frac{\pi}{2})$

$\cos(-\frac{\pi}{2}) = 0$

$x < -\frac{\pi}{2}$, $\cos x < 0.$

$\sin x < 0.$

$x - \tan x > 0.$



$x - \tan x < 0.$

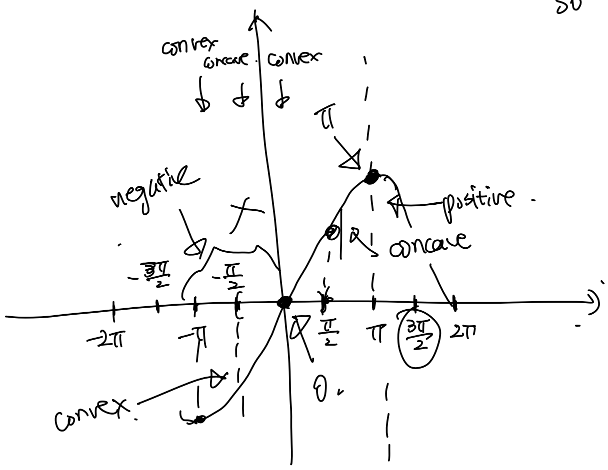
$x < \tan x.$

$\boxed{Si''(x) = \frac{x\cos x - \sin x}{x^2}}$

if $x > \frac{\pi}{2}$, then $\cos x < 0$,

so $x\cos x - \sin x > 0$ $\quad$ $x\cos x - \sin x < 0$.

is the same as $\underline{x < \tan x}$



$Si''(x) > 0$. then

$\underline{x\cos x - \sin x > 0}$. (convex).

convex

$\begin{cases} \cos x > 0. \ (-\frac{\pi}{2} < x < \frac{\pi}{2}) & \underline{x > \tan x} : (-\frac{\pi}{2}, 0) \\ \cos x < 0. \ (x < -\frac{\pi}{2} \text{ or } x > \frac{\pi}{2}) & x < \tan x. \ (-\pi, -\frac{\pi}{2}) \end{cases}$

convex.

$\begin{cases} \frac{\sin x}{x} > 0 \\ \text{as } -\pi < x < \pi \end{cases}$

$\underline{\int_0^\pi \frac{\sin t}{t}dt > 0.90}$.

$\int_0^x \frac{\sin t}{t}dt = 1$.

$\int_0^{\frac{\pi}{2}} \frac{\sin t}{t}dt < 1$ (>1)

concave: $\underline{(0, \frac{\pi}{2}) \& (\frac{\pi}{2}, \pi)}$

$(0, \pi)$

$(0, \frac{\pi}{2})$.

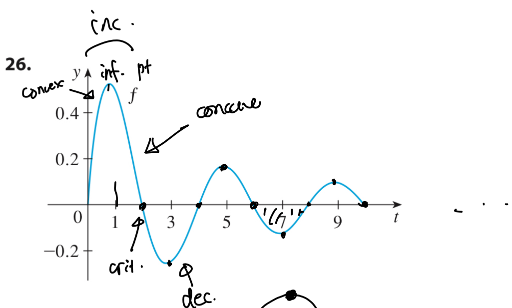
$\frac{\pi}{4}$.

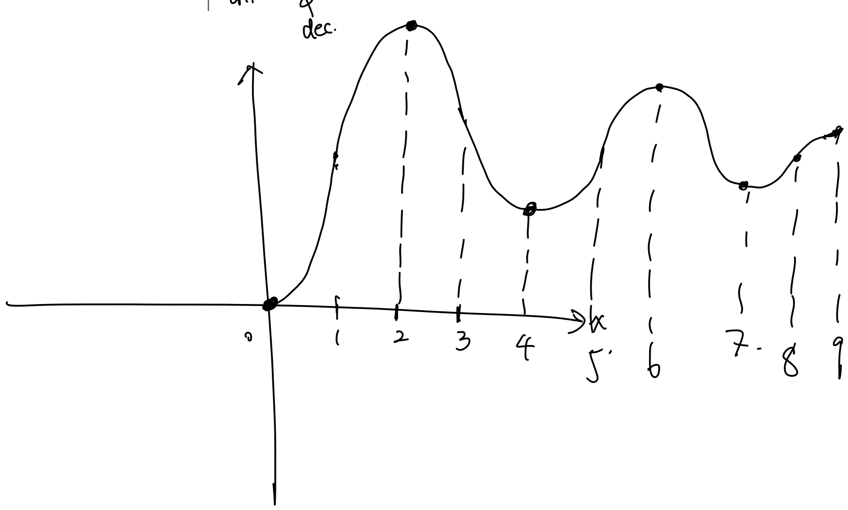
$\int \frac{\sin t}{t}dt \le \int f(t)dt$

$\int g(t)dt$

26. $f = g'$

$g(0) = 0$.





$$\int_1^2 \sqrt{4-x^2}\,dx = 2\int_1^2 \sqrt{1-\left(\frac{x}{2}\right)^2}\,dx$$

$y=\frac{x}{2}$, $x=2y$.

$1 \le x \le 2$

$\frac{1}{2} \le y = \frac{x}{2} \le 1$

$$= 4\int_{\frac{1}{2}}^{1} \sqrt{1-y^2}\,dy.$$

$y = \sin x$ $\quad y = \cos x$

$\frac{\pi}{6} \le x \le \frac{\pi}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$.

$y = -\frac{x}{2}$.

$-1 \le y \le -\frac{1}{2}$.

$$= 4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \underbrace{\sqrt{1-\sin^2 x}}_{\cos x}\,\cos x\,dx = 4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2 x\,dx.$$

$$\int_{-\frac{1}{2}}^{-1} = -\int_{-1}^{-\frac{1}{2}}.$$

$$= 4\left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right)\Bigg|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$\sin^2 x + \cos^2 x = 1$.

$\sqrt{1-x^2}$ $\quad x = \sin y \,/\, \cos y$

$$= 4\left(\underbrace{\frac{1}{2}\left(\frac{\pi}{2}-\frac{\pi}{8}\right)}_{\frac{3\pi}{8}} + \frac{1}{4}\left(0-\frac{\sqrt{3}}{2}\right)\right)$$

$\sqrt{1-\sin^2 y} \rightsquigarrow \cos y \,/\, -\cos y$ $\quad = \frac{3\pi}{4} - \frac{\sqrt{3}}{2}$.