Name: ______________________________ **MATH 125**
Date: 09/14 **Quiz 2A**

Problem 1. Find the limit $\lim_{x\to 1} \frac{1}{1-x} - \frac{2}{1-x^2}$.(10 points)

Solution. We have

$$\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{2}{1-x^2}\right) = \lim_{x\to 1}\left(\frac{1}{1-x}-\frac{2}{(1+x)(1-x)}\right) = \lim_{x\to 1}\frac{(1+x)-2}{(1+x)(1-x)} = \lim_{x\to 1}\frac{x-1}{(1+x)(1-x)}$$
$$= \lim_{x\to 1}\frac{-1}{1+x} = -\frac{1}{2}.$$ ◇

Problem 2. Find the limit $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$.(10 points)
[Hint: $\cos x = \sin(\frac{\pi}{2} - x)$.]

Solution. We have

$$\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2} = \lim_{x\to 1}\frac{(1-x)\sin\frac{\pi x}{2}}{\cos\frac{\pi x}{2}} = \lim_{x\to 1}\frac{(1-x)\sin\frac{\pi x}{2}}{\sin(\frac{\pi}{2}-\frac{\pi x}{2})}$$
$$= \lim_{x\to 1}\frac{1-x}{\sin\frac{\pi}{2}(1-x)}\cdot\sin\frac{\pi x}{2} = \lim_{x\to 1}\frac{2}{\pi}\frac{\frac{\pi}{2}(1-x)}{\sin(\frac{\pi}{2}(1-x))}\lim_{x\to 1}\sin\frac{\pi x}{2}$$
$$= \frac{2}{\pi}\cdot 1 = \frac{2}{\pi}.$$ ◇

Problem 3. Show that there is a root of the equation $x^3 - 3x + 1 = 0$ in the interval $(-2, -1)$.(10 points)

Solution. Let $f(x) = x^3 - 3x + 1$, then we have $f(x)$ is continuous in the interval $[-2, -1]$ and $f(-2) = (-2)^3 - 3\times(-2) + 1 = -8 + 6 + 1 = -1$, and $f(-1) = (-1)^3 - 3\times(-1) + 1 = -1 + 3 + 1 = 3$, so we know that $f(-2) < 0$ and $f(-1) > 0$, hence by the intermediate value theorem, f has a root in the interval $(-2, -1)$. ◇

Final Score: ______________

Name: ______________________________ **MATH 125**

Date: 09/14 **Quiz 2B**

Problem 1. Find the limit $\lim_{x\to\frac{1}{2}} \frac{1}{1-2x} - \frac{3}{2-2x-4x^2}$.(10 points)

Solution. Note that

$$\lim_{x\to\frac{1}{2}} \left(\frac{1}{1-2x} - \frac{3}{2-2x-4x^2}\right) = \lim_{x\to\frac{1}{2}} \left(\frac{1}{1-2x} - \frac{3}{2(1-2x)(1+x)}\right) = \lim_{x\to\frac{1}{2}} \frac{2(1+x)-3}{2(1-2x)(1+x)}$$
$$= \lim_{x\to\frac{1}{2}} \frac{1}{2(1+x)} = -\frac{1}{3}. \qquad \diamond$$

Problem 2. Find the limit $\lim_{x\to-1} (x+1)\tan\frac{\pi x}{2}$.(10 points)
[Hint: $\cos x = -\sin(\frac{\pi}{2}+x)$.]

Solution.

$$\lim_{x\to-1}(x+1)\tan\frac{\pi x}{2} = \lim_{x\to-1}\frac{(x+1)\sin\frac{\pi x}{2}}{\cos\frac{\pi x}{2}} = \lim_{x\to-1}\frac{(x+1)\sin\frac{\pi x}{2}}{-\sin(\frac{\pi}{2}+\frac{\pi x}{2})}$$
$$= -\lim_{x\to-1}\frac{x+1}{\sin\frac{\pi}{2}(1+x)}\lim_{x\to1}\sin\frac{\pi x}{2} = -\lim_{x\to-1}\frac{2}{\pi}\frac{\frac{\pi}{2}(x+1)}{\sin\frac{\pi}{2}(1+x)}\lim_{x\to-1}\sin\frac{\pi}{2}x$$
$$= -\frac{2}{\pi}\cdot(-1) = \frac{2}{\pi}. \qquad \diamond$$

Problem 3. Show that there's a root of the equation $\sin 2x = x^2 - 2$ in the interval $(0,2)$.(10 points)

Solution. Let $f(x) = \sin 2x - x^2 + 2$, then a root of the equation $\sin 2x = x^2 - 2$ would be a root of the function $f(x)$. Now f is continuous in the interval $[0,2]$, $f(0) = 2 > 0$ and $f(2) = \sin 4 - 4 + 2 = \sin 4 - 2 < 0$, hence by the intermediate value theorem, $f(x)$ has a root in the interval $(0,2)$. That is, the equation $\sin 2x = x^2 - 2$ has a root in the interval $(0,2)$. $\diamond$

Final Score: ______________