Name: ______________________ **MATH 125**
Date: 09/28 **Quiz 4A**

Problem 1. Differentiate the function $y = \dfrac{x^2}{2 - \sin x}$.

Solution.

$$\begin{aligned} y' = \left(\frac{x^2}{2-\sin x}\right)' &= \frac{(x^2)'(2-\sin x) - (2-\sin x)'x^2}{(2-\sin x)^2} = \frac{2x(2-\sin x) - x^2(-\cos x)}{(2-\sin x)^2} \\ &= \frac{x^2\cos x + 4x - 2x\sin x}{(2-\sin x)^2}. \end{aligned}$$

$\diamond$

Problem 2. Find the limit $\lim_{x\to+\infty} \cos\frac{1}{x}\sin\frac{1}{x}$ using squeeze theorem.

Solution. We use the inequality $-1 \le \cos\frac{1}{x} \le 1$. Multiply both sides by $\sin\frac{1}{x}$(and because x tends to $+\infty$, $\sin\frac{1}{x}$ is positive near $+\infty$), we get

$$-\sin\frac{1}{x} \le \cos\frac{1}{x}\sin\frac{1}{x} \le \sin\frac{1}{x}.$$

Taking the limit as $x \to +\infty$, we get $\lim_{x\to+\infty}\sin\frac{1}{x} = \lim_{x\to+\infty}\left(-\sin\frac{1}{x}\right) = 0$, so by the squeeze theorem, we have $\lim_{x\to+\infty}\cos\frac{1}{x}\sin\frac{1}{x} = 0$. $\diamond$

Final Score: ______________

Name: ______________________________ **MATH 125**
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Problem 1. Differentiate the function $y = \dfrac{\sin x}{2 - x^2}$.

Solution.

$$\begin{aligned} y' &= \left(\frac{\sin x}{2-x^2}\right)' = \frac{(\sin x)'(2-x^2) - (\sin x)(2-x^2)'}{(2-x^2)^2} = \frac{(2-x^2)\cos x - (-2x)\sin x}{(2-x^2)^2} \\ &= \frac{2\cos x + 2x\sin x - x^2\cos x}{(2-x^2)^2}. \end{aligned}$$

◇

Problem 2. Find the limit $\displaystyle\lim_{x\to+\infty} \sin x \tan\frac{1}{x}$ using squeeze theorem.

Solution. We have the inequality $-1 \le \sin x \le 1$, and because $\tan\frac{1}{x}$ is positive as $x \to +\infty$, we have

$$-\tan\frac{1}{x} \le \sin x \tan\frac{1}{x} \le \tan\frac{1}{x}.$$

Taking the limit as $x \to +\infty$, we get that $\displaystyle\lim_{x\to+\infty}(-\tan\frac{1}{x}) = \lim_{x\to+\infty}\tan\frac{1}{x} = 0$. Therefore by the squeeze theorem, $\lim_{x\to+\infty} \sin x \tan\frac{1}{x} = 0$. ◇

Final Score: ______________